Math 155, ***Lecture Notes- Bonds*** **Name____________**

Section 9.5 *Alternating Series*

Up until now, we have been studying series with positive terms. In this section we will study series that have both positive and negative terms. In particular, we will start by consider series that have terms that alternate in and we will call these **alternating series**. In order to prove convergence, or divergence for these series, we will still need to use our knowledge of the convergence and divergence characteristics of geometric series, harmonic series, *p*-series, and telescoping series.

Alternating series can be written in two ways, either the even terms are negative, or the odd terms are negative. Below, we have a convergence test for alternating series:

THEOREM 9.14 Alternating Series Test

Let $a_n > 0$. The alternating series

$$\sum_{n=1}^{\infty} (-1)^n a_n \quad \text{and} \quad \sum_{n=1}^{\infty} (-1)^{n+1} a_n$$

converge if the following two conditions are met.

1. $\lim_{n\to\infty} a_n = 0$ **2.** $a_{n+1} \le a_n$, for all n

Ex. 1: Determine the convergence or divergence of the series: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

Ex. 2: Determine the convergence or divergence of the series: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{n^2+1}$

Ex. 3: Determine the convergence or divergence of the series: $\sum_{n=1}^{\infty} \frac{\left(-1\right)^{n+1} n^2}{n^2+5}$

Ex. 4: Consider: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$

THEOREM 9.15 Alternating Series Remainder

If a convergent alternating series satisfies the condition $a_{n+1} \leq a_n$, then the absolute value of the remainder R_N involved in approximating the sum S by S_N is less than (or equal to) the first neglected term. That is,

$$|S - S_N| = |R_N| \leq a_{N+1}.$$

Ex. 5: (a) Use *Theorem 9.15* to determine the number of terms required to approximate the sum of the convergent series with an error of less than 0.001.
(b) Use a graphing utility approximate the sum of the series with an error if less than 0.001

Consider: $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} = \cos(1)$

THEOREM 9.16 Absolute Convergence

If the series $\Sigma|a_n|$ converges, then the series $\Sigma\, a_n$ also converges.

Definitions of Absolute and Conditional Convergence

1. $\Sigma\, a_n$ is **absolutely convergent** if $\Sigma\, |a_n|$ converges.
2. $\Sigma\, a_n$ is **conditionally convergent** if $\Sigma\, a_n$ converges but $\Sigma\, |a_n|$ diverges.

Ex. 6: Determine the convergence or divergence of the series: $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+4}}$

Ex. 7: Determine the convergence or divergence of the series: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\sqrt{n}}$